

6

Circular Motion and Other Applications of Newton's Laws

CHAPTER OUTLINE

- 6.1 Newton's Second Law Applied to Uniform Circular Motion
- 6.2 Nonuniform Circular Motion
- 6.3 Motion in Accelerated Frames
- 6.4 Motion in the Presence of Resistive Forces
- 6.5 Numerical Modeling in Particle Dynamics

ANSWERS TO QUESTIONS

- Q6.1** Mud flies off a rapidly spinning tire because the resultant force is not sufficient to keep it moving in a circular path. In this case, the force that plays a major role is the adhesion between the mud and the tire.
- Q6.2** The spring will stretch. In order for the object to move in a circle, the force exerted on the object by the spring must have a size of $\frac{mv^2}{r}$. Newton's third law says that the force exerted on the object by the spring has the same size as the force exerted by the object on the spring. It is the force exerted on the spring that causes the spring to stretch.
- Q6.3** Driving in a circle at a constant speed requires a centripetal acceleration but no tangential acceleration.
- Q6.4** (a) The object will move in a circle at a constant speed.
(b) The object will move in a straight line at a changing speed.
- Q6.5** The speed changes. The tangential force component causes tangential acceleration.
- Q6.6** Consider the force required to keep a rock in the Earth's crust moving in a circle. The size of the force is proportional to the radius of the circle. If that rock is at the Equator, the radius of the circle through which it moves is about 6400 km. If the rock is at the north pole, the radius of the circle through which it moves is zero!
- Q6.7** Consider standing on a bathroom scale. The resultant force on you is your actual weight minus the normal force. The scale reading shows the size of the normal force, and is your 'apparent weight.' If you are at the North or South Pole, it can be precisely equal to your actual weight. If you are at the equator, your apparent weight must be less, so that the resultant force on you can be a downward force large enough to cause your centripetal acceleration as the Earth rotates.
- Q6.8** A torque is exerted by the thrust force of the water times the distance between the nozzles.

- Q6.9** I would not accept that statement for two reasons. First, to be “beyond the pull of gravity,” one would have to be infinitely far away from all other matter. Second, astronauts in orbit are moving in a circular path. It is the gravitational pull of Earth on the astronauts that keeps them in orbit. In the space shuttle, just above the atmosphere, gravity is only slightly weaker than at the Earth’s surface. Gravity does its job most clearly on an orbiting spacecraft, because the craft feels no other forces and is in free fall.
- Q6.10** This is the same principle as the centrifuge. All the material inside the cylinder tends to move along a straight-line path, but the walls of the cylinder exert an inward force to keep everything moving around in a circular path.
- Q6.11** The ball would not behave as it would when dropped on the Earth. As the astronaut holds the ball, she and the ball are moving with the same angular velocity. The ball, however, being closer to the center of rotation, is moving with a slower tangential velocity. Once the ball is released, it acts according to Newton’s first law, and simply drifts with constant velocity in the original direction of its velocity when released—it is no longer “attached” to the rotating space station. Since the ball follows a straight line and the astronaut follows a circular path, it will appear to the astronaut that the ball will “fall to the floor”. But other dramatic effects will occur. Imagine that the ball is held so high that it is just slightly away from the center of rotation. Then, as the ball is released, it will move very slowly along a straight line. Thus, the astronaut may make several full rotations around the circular path before the ball strikes the floor. This will result in three obvious variations with the Earth drop. First, the time to fall will be much larger than that on the Earth, even though the feet of the astronaut are pressed into the floor with a force that suggests the same force of gravity as on Earth. Second, the ball may actually appear to bob up and down if several rotations are made while it “falls”. As the ball moves in a straight line while the astronaut rotates, sometimes she is on the side of the circle on which the ball is moving toward her and other times she is on the other side, where the ball is moving away from her. The third effect is that the ball will not drop straight down to her feet. In the extreme case we have been imagining, it may actually strike the surface while she is on the opposite side, so it looks like it ended up “falling up”. In the least extreme case, in which only a portion of a rotation is made before the ball strikes the surface, the ball will appear to move backward relative to the astronaut as it falls.
- Q6.12** The water has inertia. The water tends to move along a straight line, but the bucket pulls it in and around in a circle.
- Q6.13** There is no such force. If the passenger slides outward across the slippery car seat, it is because the passenger is moving forward in a straight line while the car is turning under him. If the passenger pushes hard against the outside door, the door is exerting an inward force on him. No object is exerting an outward force on him, but he should still buckle his seatbelt.
- Q6.14** Blood pressure cannot supply the force necessary both to balance the gravitational force and to provide the centripetal acceleration, to keep blood flowing up to the pilot’s brain.
- Q6.15** The person in the elevator is in an accelerating reference frame. The apparent acceleration due to gravity, “ g ,” is changed inside the elevator. “ g ” = $g \pm a$
- Q6.16** When you are not accelerating, the normal force and your weight are equal in size. Your body interprets the force of the floor pushing up on you as your weight. When you accelerate in an elevator, this normal force changes so that you accelerate with the elevator. In free fall, you are never weightless since the Earth’s gravity and your mass do not change. It is the normal force—your apparent weight—that is zero.

- Q6.17** From the proportionality of the drag force to the speed squared and from Newton's second law, we derive the equation that describes the motion of the skydiver:

$$m \frac{dv_y}{dt} = mg - \frac{D\rho A}{2} v_y^2$$

where D is the coefficient of drag of the parachutist, and A is the projected area of the parachutist's body. At terminal speed,

$$a_y = \frac{dv_y}{dt} = 0 \text{ and } V_T \left(\frac{2mg}{D\rho A} \right)^{1/2}.$$

When the parachute opens, the coefficient of drag D and the effective area A both increase, thus reducing the speed of the skydiver.

Modern parachutes also add a third term, lift, to change the equation to

$$m \frac{dv_y}{dt} = mg - \frac{D\rho A}{2} v_y^2 - \frac{L\rho A}{2} v_x^2$$

where v_y is the vertical velocity, and v_x is the horizontal velocity. The effect of lift is clearly seen in the "paraplane," an ultralight airplane made from a fan, a chair, and a parachute.

- Q6.18** The larger drop has higher terminal speed. In the case of spheres, the text demonstrates that terminal speed is proportional to the square root of radius. When moving with terminal speed, an object is in equilibrium and has zero acceleration.
- Q6.19** Lower air density reduces air resistance, so a tank-truck-load of fuel takes you farther.
- Q6.20** Suppose the rock is moving rapidly when it enters the water. The speed of the rock decreases until it reaches terminal velocity. The acceleration, which is upward, decreases to zero as the rock approaches terminal velocity.
- Q6.21** The thesis is false. The moment of decay of a radioactive atomic nucleus (for example) cannot be predicted. Quantum mechanics implies that the future is indeterminate. On the other hand, our sense of free will, of being able to make choices for ourselves that can appear to be random, may be an illusion. It may have nothing to do with the subatomic randomness described by quantum mechanics.

SOLUTIONS TO PROBLEMS

Section 6.1 Newton's Second Law Applied to Uniform Circular Motion

P6.1 $m = 3.00 \text{ kg}$, $r = 0.800 \text{ m}$. The string will break if the tension exceeds the weight corresponding to 25.0 kg , so

$$T_{\max} = Mg = 25.0(9.80) = 245 \text{ N}.$$

When the 3.00 kg mass rotates in a horizontal circle, the tension causes the centripetal acceleration,

$$\text{so } T = \frac{mv^2}{r} = \frac{(3.00)v^2}{0.800}.$$

$$\text{Then } v^2 = \frac{rT}{m} = \frac{(0.800)T}{3.00} \leq \frac{(0.800)T_{\max}}{3.00} = \frac{0.800(245)}{3.00} = 65.3 \text{ m}^2/\text{s}^2$$

$$\text{and } 0 \leq v \leq \sqrt{65.3}$$

$$\text{or } \boxed{0 \leq v \leq 8.08 \text{ m/s}}.$$

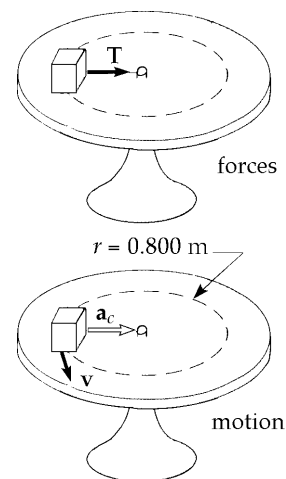


FIG. P6.1

P6.2 In $\sum F = m \frac{v^2}{r}$, both m and r are unknown but remain constant. Therefore, $\sum F$ is proportional to v^2 and increases by a factor of $\left(\frac{18.0}{14.0}\right)^2$ as v increases from 14.0 m/s to 18.0 m/s . The total force at the higher speed is then

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 (130 \text{ N}) = 215 \text{ N}.$$

Symbolically, write $\sum F_{\text{slow}} = \left(\frac{m}{r}\right)(14.0 \text{ m/s})^2$ and $\sum F_{\text{fast}} = \left(\frac{m}{r}\right)(18.0 \text{ m/s})^2$.

Dividing gives $\frac{\sum F_{\text{fast}}}{\sum F_{\text{slow}}} = \left(\frac{18.0}{14.0}\right)^2$, or

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 \sum F_{\text{slow}} = \left(\frac{18.0}{14.0}\right)^2 (130 \text{ N}) = \boxed{215 \text{ N}}.$$

This force must be horizontally inward to produce the driver's centripetal acceleration.

P6.3 (a) $F = \frac{mv^2}{r} = \frac{(9.11 \times 10^{-31} \text{ kg})(2.20 \times 10^6 \text{ m/s})^2}{0.530 \times 10^{-10} \text{ m}} = \boxed{8.32 \times 10^{-8} \text{ N inward}}$

(b) $a = \frac{v^2}{r} = \frac{(2.20 \times 10^6 \text{ m/s})^2}{0.530 \times 10^{-10} \text{ m}} = \boxed{9.13 \times 10^{22} \text{ m/s}^2 \text{ inward}}$

P6.4 Neglecting relativistic effects. $F = ma_c = \frac{mv^2}{r}$

$$F = (2 \times 1.661 \times 10^{-27} \text{ kg}) \frac{(2.998 \times 10^7 \text{ m/s})^2}{(0.480 \text{ m})} = \boxed{6.22 \times 10^{-12} \text{ N}}$$

P6.5 (a) $\boxed{\text{static friction}}$

(b) $ma\hat{\mathbf{i}} = f\hat{\mathbf{i}} + n\hat{\mathbf{j}} + mg(-\hat{\mathbf{j}})$

$$\sum F_y = 0 = n - mg$$

thus $n = mg$ and $\sum F_r = m \frac{v^2}{r} = f = \mu n = \mu mg$.

Then $\mu = \frac{v^2}{rg} = \frac{(50.0 \text{ cm/s})^2}{(30.0 \text{ cm})(980 \text{ cm/s}^2)} = \boxed{0.0850}$.

P6.6 (a) $\sum F_y = ma_y, mg_{\text{moon}} \text{ down} = \frac{mv^2}{r} \text{ down}$

$$v = \sqrt{g_{\text{moon}} r} = \sqrt{(1.52 \text{ m/s}^2)(1.7 \times 10^6 \text{ m} + 100 \times 10^3 \text{ m})} = \boxed{1.65 \times 10^3 \text{ m/s}}$$

(b) $v = \frac{2\pi r}{T}, T = \frac{2\pi(1.8 \times 10^6 \text{ m})}{1.65 \times 10^3 \text{ m/s}} = \boxed{6.84 \times 10^3 \text{ s}} = 1.90 \text{ h}$

P6.7 $n = mg$ since $a_y = 0$

The force causing the centripetal acceleration is the frictional force f .

From Newton's second law $f = ma_c = \frac{mv^2}{r}$.

But the friction condition is $f \leq \mu_s n$

i.e., $\frac{mv^2}{r} \leq \mu_s mg$

$$v \leq \sqrt{\mu_s rg} = \sqrt{0.600(35.0 \text{ m})(9.80 \text{ m/s}^2)} \quad v \leq \boxed{14.3 \text{ m/s}}$$

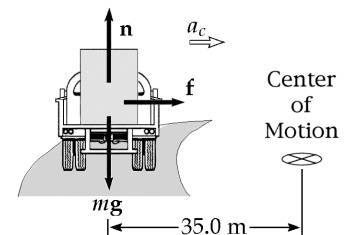


FIG. P6.7

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P6.8
$$a = \frac{v^2}{r} = \frac{\left[(86.5 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \right]^2}{61.0 \text{ m}} \left(\frac{1 \text{ g}}{9.80 \text{ m/s}^2} \right) = \boxed{0.966 \text{ g}}$$

P6.9 $T \cos 5.00^\circ = mg = (80.0 \text{ kg})(9.80 \text{ m/s}^2)$

(a) $T = 787 \text{ N}; \mathbf{T} = \boxed{(68.6 \text{ N})\hat{\mathbf{i}} + (784 \text{ N})\hat{\mathbf{j}}}$

(b) $T \sin 5.00^\circ = ma_c: \boxed{a_c = 0.857 \text{ m/s}^2}$ toward the center of the circle.

The length of the wire is unnecessary information. We could, on the other hand, use it to find the radius of the circle, the speed of the bob, and the period of the motion.

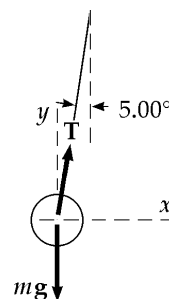


FIG. P6.9

P6.10 (b) $v = \frac{235 \text{ m}}{36.0 \text{ s}} = \boxed{6.53 \text{ m/s}}$

The radius is given by $\frac{1}{4} 2\pi r = 235 \text{ m}$

$$r = 150 \text{ m}$$

(a)
$$\begin{aligned} \mathbf{a}_r &= \left(\frac{v^2}{r} \right) \text{ toward center} \\ &= \frac{(6.53 \text{ m/s})^2}{150 \text{ m}} \text{ at } 35.0^\circ \text{ north of west} \\ &= (0.285 \text{ m/s}^2) (\cos 35.0^\circ (-\hat{\mathbf{i}}) + \sin 35.0^\circ \hat{\mathbf{j}}) \\ &= \boxed{-0.233 \text{ m/s}^2 \hat{\mathbf{i}} + 0.163 \text{ m/s}^2 \hat{\mathbf{j}}} \end{aligned}$$

(c)
$$\begin{aligned} \bar{\mathbf{a}} &= \frac{(\mathbf{v}_f - \mathbf{v}_i)}{t} \\ &= \frac{(6.53 \text{ m/s} \hat{\mathbf{j}} - 6.53 \text{ m/s} \hat{\mathbf{i}})}{36.0 \text{ s}} \\ &= \boxed{-0.181 \text{ m/s}^2 \hat{\mathbf{i}} + 0.181 \text{ m/s}^2 \hat{\mathbf{j}}} \end{aligned}$$

***P6.11** $F_g = mg = (4 \text{ kg})(9.8 \text{ m/s}^2) = 39.2 \text{ N}$

$$\sin \theta = \frac{1.5 \text{ m}}{2 \text{ m}}$$

$$\theta = 48.6^\circ$$

$$r = (2 \text{ m}) \cos 48.6^\circ = 1.32 \text{ m}$$

$$\sum F_x = ma_x = \frac{mv^2}{r}$$

$$T_a \cos 48.6^\circ + T_b \cos 48.6^\circ = \frac{(4 \text{ kg})(6 \text{ m/s})^2}{1.32 \text{ m}}$$

$$T_a + T_b = \frac{109 \text{ N}}{\cos 48.6^\circ} = 165 \text{ N}$$

$$\sum F_y = ma_y$$

$$+T_a \sin 48.6^\circ - T_b \sin 48.6^\circ - 39.2 \text{ N} = 0$$

$$T_a - T_b = \frac{39.2 \text{ N}}{\sin 48.6^\circ} = 52.3 \text{ N}$$

(a) To solve simultaneously, we add the equations in T_a and T_b :

$$T_a + T_b + T_a - T_b = 165 \text{ N} + 52.3 \text{ N}$$

$$T_a = \frac{217 \text{ N}}{2} = \boxed{108 \text{ N}}$$

(b) $T_b = 165 \text{ N} - T_a = 165 \text{ N} - 108 \text{ N} = \boxed{56.2 \text{ N}}$

***P6.12** $a_c = \frac{v^2}{r}$. Let f represent the rotation rate. Each revolution carries each bit of metal through distance $2\pi r$, so $v = 2\pi rf$ and

$$a_c = \frac{v^2}{r} = 4\pi^2 rf^2 = 100 g.$$

A smaller radius implies smaller acceleration. To meet the criterion for each bit of metal we consider the minimum radius:

$$f = \left(\frac{100 g}{4\pi^2 r} \right)^{1/2} = \left(\frac{100 \cdot 9.8 \text{ m/s}^2}{4\pi^2 (0.021 \text{ m})} \right)^{1/2} = 34.4 \frac{1}{\text{s}} \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = \boxed{2.06 \times 10^3 \text{ rev/min}}.$$

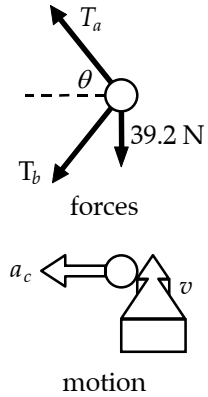


FIG. P6.11

Section 6.2 Nonuniform Circular Motion

P6.13 $M = 40.0 \text{ kg}$, $R = 3.00 \text{ m}$, $T = 350 \text{ N}$

$$\begin{aligned} \text{(a)} \quad \sum F &= 2T - Mg = \frac{Mv^2}{R} \\ v^2 &= (2T - Mg) \left(\frac{R}{M} \right) \\ v^2 &= [700 - (40.0)(9.80)] \left(\frac{3.00}{40.0} \right) = 23.1 \text{ (m}^2/\text{s}^2) \\ \boxed{v = 4.81 \text{ m/s}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad n - Mg &= F = \frac{Mv^2}{R} \\ n &= Mg + \frac{Mv^2}{R} = 40.0 \left(9.80 + \frac{23.1}{3.00} \right) = \boxed{700 \text{ N}} \end{aligned}$$

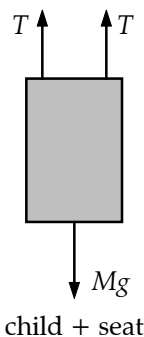


FIG. P6.13(a)

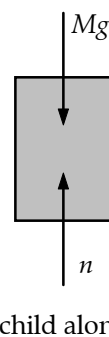


FIG. P6.13(b)

P6.14 (a) Consider the forces acting on the system consisting of the child *and* the seat:

$$\begin{aligned} \sum F_y &= ma_y \Rightarrow 2T - mg = m \frac{v^2}{R} \\ v^2 &= R \left(\frac{2T}{m} - g \right) \\ v &= \sqrt{R \left(\frac{2T}{m} - g \right)} \end{aligned}$$

(b) Consider the forces acting on the child alone:

$$\sum F_y = ma_y \Rightarrow n = m \left(g + \frac{v^2}{R} \right)$$

and from above, $v^2 = R \left(\frac{2T}{m} - g \right)$, so

$$n = m \left(g + \frac{2T}{m} - g \right) = \boxed{2T}.$$

P6.15 Let the tension at the lowest point be T .

$$\begin{aligned} \sum F &= ma: \quad T - mg = ma_c = \frac{mv^2}{r} \\ T &= m \left(g + \frac{v^2}{r} \right) \\ T &= (85.0 \text{ kg}) \left[9.80 \text{ m/s}^2 + \frac{(8.00 \text{ m/s})^2}{10.0 \text{ m}} \right] = 1.38 \text{ kN} > 1000 \text{ N} \end{aligned}$$

He doesn't make it across the river because the vine breaks.

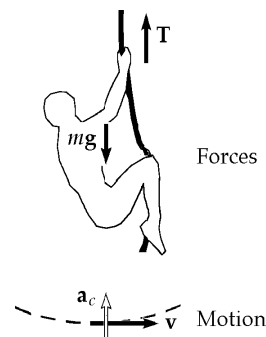


FIG. P6.15

P6.16 (a) $a_c = \frac{v^2}{r} = \frac{(4.00 \text{ m/s})^2}{12.0 \text{ m}} = \boxed{1.33 \text{ m/s}^2}$

(b) $a = \sqrt{a_c^2 + a_t^2}$
 $a = \sqrt{(1.33)^2 + (1.20)^2} = \boxed{1.79 \text{ m/s}^2}$
 at an angle $\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right) = \boxed{48.0^\circ \text{ inward}}$

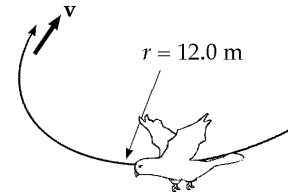


FIG. P6.16

P6.17 $\sum F_y = \frac{mv^2}{r} = mg + n$

But $n = 0$ at this minimum speed condition, so

$$\frac{mv^2}{r} = mg \Rightarrow v = \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(1.00 \text{ m})} = \boxed{3.13 \text{ m/s}}.$$

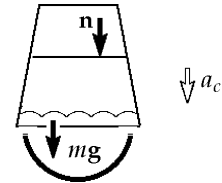


FIG. P6.17

P6.18 At the top of the vertical circle,

$$T = m \frac{v^2}{R} - mg$$

or $T = (0.400) \frac{(4.00)^2}{0.500} - (0.400)(9.80) = \boxed{8.88 \text{ N}}$

P6.19 (a) $v = 20.0 \text{ m/s}$,
 $n =$ force of track on roller coaster, and
 $R = 10.0 \text{ m}$.

$$\sum F = \frac{Mv^2}{R} = n - Mg$$

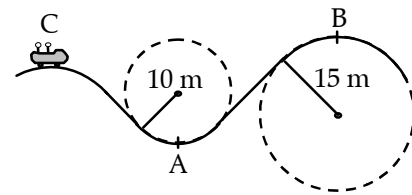


FIG. P6.19

From this we find

$$n = Mg + \frac{Mv^2}{R} = (500 \text{ kg})(9.80 \text{ m/s}^2) + \frac{(500 \text{ kg})(20.0 \text{ m/s})^2}{10.0 \text{ m}}$$

$$n = 4900 \text{ N} + 20000 \text{ N} = \boxed{2.49 \times 10^4 \text{ N}}$$

(b) At B, $n - Mg = -\frac{Mv^2}{R}$

The max speed at B corresponds to

$$n = 0$$

$$-Mg = -\frac{Mv_{\max}^2}{R} \Rightarrow v_{\max} = \sqrt{Rg} = \sqrt{15.0(9.80)} = \boxed{12.1 \text{ m/s}}$$

P6.20 (a) $a_c = \frac{v^2}{r}$ $r = \frac{v^2}{a_c} = \frac{(13.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{8.62 \text{ m}}$

- (b) Let n be the force exerted by the rail.

Newton's law gives

$$Mg + n = \frac{Mv^2}{r}$$

$$n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = \boxed{Mg, \text{ downward}}$$

(c) $a_c = \frac{v^2}{r}$ $a_c = \frac{(13.0 \text{ m/s})^2}{20.0 \text{ m}} = \boxed{8.45 \text{ m/s}^2}$

If the force exerted by the rail is n_1

then $n_1 + Mg = \frac{Mv^2}{r} = Ma_c$

$$n_1 = M(a_c - g) \text{ which is } < 0 \text{ since } a_c = 8.45 \text{ m/s}^2$$

Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars. To be safe we must require n_1 to be positive.

Then $a_c > g$. We need

$$\frac{v^2}{r} > g \text{ or } v > \sqrt{rg} = \sqrt{(20.0 \text{ m})(9.80 \text{ m/s}^2)}, v > 14.0 \text{ m/s}.$$

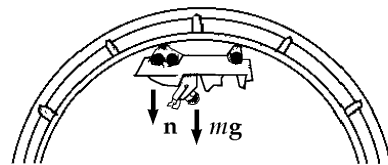


FIG. P6.20

Section 6.3 Motion in Accelerated Frames

P6.21 (a) $\sum F_x = Ma, a = \frac{T}{M} = \frac{18.0 \text{ N}}{5.00 \text{ kg}} = \boxed{3.60 \text{ m/s}^2}$

to the right.

- (b) If $v = \text{const}$, $a = 0$, so $\boxed{T = 0}$ (This is also an equilibrium situation.)

- (c) Someone in the car (noninertial observer) claims that the forces on the mass along x are T and a fictitious force $(-Ma)$. Someone at rest outside the car (inertial observer) claims that T is the only force on M in the x -direction.

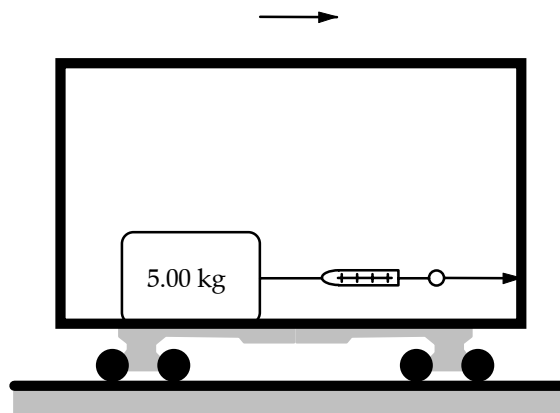


FIG. P6.21

- *P6.22** We adopt the view of an inertial observer. If it is on the verge of sliding, the cup is moving on a circle with its centripetal acceleration caused by friction.

$$\begin{aligned}\sum F_y = ma_y: \quad +n - mg &= 0 \\ \sum F_x = ma_x: \quad f = \frac{mv^2}{r} &= \mu_s n = \mu_s mg\end{aligned}$$

$$v = \sqrt{\mu_s g r} = \sqrt{0.8(9.8 \text{ m/s}^2)(30 \text{ m})} = \boxed{15.3 \text{ m/s}}$$

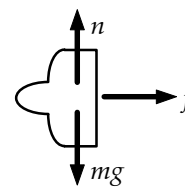


FIG. P6.22

If you go too fast the cup will begin sliding straight across the dashboard to the left.

- P6.23** The only forces acting on the suspended object are the force of gravity mg and the force of tension T , as shown in the free-body diagram. Applying Newton's second law in the x and y directions,

$$\sum F_x = T \sin \theta = ma \quad (1)$$

$$\sum F_y = T \cos \theta - mg = 0$$

or $T \cos \theta = mg \quad (2)$

- (a) Dividing equation (1) by (2) gives

$$\tan \theta = \frac{a}{g} = \frac{3.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} = 0.306.$$

Solving for θ , $\theta = \boxed{17.0^\circ}$

- (b) From Equation (1),

$$T = \frac{ma}{\sin \theta} = \frac{(0.500 \text{ kg})(3.00 \text{ m/s}^2)}{\sin(17.0^\circ)} = \boxed{5.12 \text{ N}}.$$

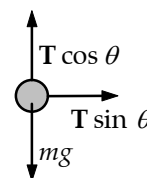


FIG. P6.23

- *P6.24** The water moves at speed

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.12 \text{ m})}{7.25 \text{ s}} = 0.104 \text{ m/s}.$$

The top layer of water feels a downward force of gravity mg and an outward fictitious force in the turntable frame of reference,

$$\frac{mv^2}{r} = \frac{m(0.104 \text{ m/s})^2}{0.12 \text{ m}} = m9.01 \times 10^{-2} \text{ m/s}^2.$$

It behaves as if it were stationary in a gravity field pointing downward and outward at

$$\tan^{-1} \frac{0.0901 \text{ m/s}^2}{9.8 \text{ m/s}^2} = \boxed{0.527^\circ}.$$

Its surface slopes upward toward the outside, making this angle with the horizontal.

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P6.25 $F_{\max} = F_g + ma = 591 \text{ N}$
 $F_{\min} = F_g - ma = 391 \text{ N}$

(a) Adding, $2F_g = 982 \text{ N}$, $F_g = \boxed{491 \text{ N}}$

(b) Since $F_g = mg$, $m = \frac{491 \text{ N}}{9.80 \text{ m/s}^2} = \boxed{50.1 \text{ kg}}$

(c) Subtracting the above equations,
 $2ma = 200 \text{ N} \quad \therefore a = \boxed{2.00 \text{ m/s}^2}$

P6.26 (a) $\sum F_r = ma_r$
 $mg = \frac{mv^2}{R} = \frac{m}{R} \left(\frac{2\pi R}{T} \right)^2$
 $g = \frac{4\pi^2 R}{T^2}$
 $T = \sqrt{\frac{4\pi^2 R}{g}} = 2\pi \sqrt{\frac{6.37 \times 10^6 \text{ m}}{9.80 \text{ m/s}^2}} = 5.07 \times 10^3 \text{ s} = \boxed{1.41 \text{ h}}$

(b) speed increase factor $= \frac{v_{\text{new}}}{v_{\text{current}}} = \frac{2\pi R}{T_{\text{new}}} \left(\frac{T_{\text{current}}}{2\pi R} \right) = \frac{T_{\text{current}}}{T_{\text{new}}} = \frac{24.0 \text{ h}}{1.41 \text{ h}} = \boxed{17.1}$

***P6.27** The car moves to the right with acceleration a . We find the acceleration of a_b of the block relative to the Earth. The block moves to the right also.

$$\begin{aligned} \sum F_y = ma_y: \quad +n - mg &= 0, \quad n = mg, \quad f = \mu_k mg \\ \sum F_x = ma_x: \quad +\mu_k mg &= ma_b, \quad a_b = \mu_k g \end{aligned}$$

The acceleration of the block relative to the car is $a_b - a = \mu_k g - a$. In this frame the block starts from rest and undergoes displacement $-\ell$ and gains speed according to

$$\begin{aligned} v_{xf}^2 &= v_{xi}^2 + 2a_x(x_f - x_i) \\ v_{xf}^2 &= 0 + 2(\mu_k g - a)(-\ell - 0) = 2\ell(a - \mu_k g). \end{aligned}$$

(a) $\boxed{v = (2\ell(a - \mu_k g))^{1/2}}$ to the left

continued on next page

(b) The time for which the box slides is given by

$$\begin{aligned}\Delta x &= \frac{1}{2}(v_{xi} + v_{xf})t \\ -\ell &= \frac{1}{2}\left[0 - (2\ell(a - \mu_k g))^{1/2}\right]t \\ t &= \left(\frac{2\ell}{a - \mu_k g}\right)^{1/2}.\end{aligned}$$

The car in the Earth frame acquires final speed $v_{xf} = v_{xi} + at = 0 + a\left(\frac{2\ell}{a - \mu_k g}\right)^{1/2}$. The speed of the box in the Earth frame is then

$$\begin{aligned}v_{be} &= v_{bc} + v_{ce} = -[2\ell(a - \mu_k g)]^{1/2} + a\left(\frac{2\ell}{a - \mu_k g}\right)^{1/2} \\ &= \frac{-(2\ell)^{1/2}(a - \mu_k g) + (2\ell)^{1/2}a}{(a - \mu_k g)^{1/2}} = \boxed{\frac{\mu_k g(2\ell)^{1/2}}{(a - \mu_k g)^{1/2}}} \\ &= \frac{\mu_k g 2\ell}{[2\ell(a - \mu_k g)]^{1/2}} = \frac{2\mu_k g \ell}{v}.\end{aligned}$$

***P6.28** Consider forces on the backpack as it slides in the Earth frame of reference.

$$\begin{aligned}\sum F_y &= ma_y: \quad +n - mg = ma, \quad n = m(g + a), \quad f_k = \mu_k m(g + a) \\ \sum F_x &= ma_x: \quad -\mu_k m(g + a) = ma_x\end{aligned}$$

The motion across the floor is described by $L = vt + \frac{1}{2}a_x t^2 = vt - \frac{1}{2}\mu_k(g + a)t^2$.

We solve for μ_k : $vt - L = \frac{1}{2}\mu_k(g + a)t^2$, $\boxed{\frac{2(vt - L)}{(g + a)t^2} = \mu_k}$.

P6.29 In an inertial reference frame, the girl is accelerating horizontally inward at

$$\frac{v^2}{r} = \frac{(5.70 \text{ m/s})^2}{2.40 \text{ m}} = 13.5 \text{ m/s}^2$$

In her own non-inertial frame, her head feels a horizontally outward fictitious force equal to its mass times this acceleration. Together this force and the weight of her head add to have a magnitude equal to the mass of her head times an acceleration of

$$\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2} = \sqrt{(9.80)^2 + (13.5)^2} \text{ m/s}^2 = 16.7 \text{ m/s}^2$$

This is larger than g by a factor of $\frac{16.7}{9.80} = 1.71$.

Thus, the force required to lift her head is larger by this factor, or the required force is

$$F = 1.71(55.0 \text{ N}) = \boxed{93.8 \text{ N}}.$$

- *P6.30** (a) The chunk is at radius $r = \frac{0.137 \text{ m} + 0.080 \text{ m}}{4} = 0.0542 \text{ m}$. Its speed is

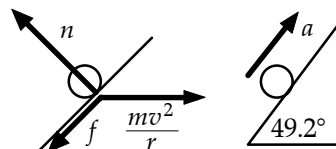
$$v = \frac{2\pi r}{T} = 2\pi(0.0542 \text{ m}) \frac{20\,000}{60 \text{ s}} = 114 \text{ m/s}$$

and its acceleration

$$a_c = \frac{v^2}{r} = \frac{(114 \text{ m/s})^2}{0.0542 \text{ m}} = \boxed{2.38 \times 10^5 \text{ m/s}^2 \text{ horizontally inward}}$$

$$= 2.38 \times 10^5 \text{ m/s}^2 \left(\frac{g}{9.8 \text{ m/s}^2} \right) = \boxed{2.43 \times 10^4 g}.$$

- (b) In the frame of the turning cone, the chunk feels a horizontally outward force of $\frac{mv^2}{r}$. In this frame its acceleration is up along the cone, at $\tan^{-1} \frac{3.3 \text{ cm}}{\frac{(13.7-8) \text{ cm}}{2}} = 49.2^\circ$.



Take the y axis perpendicular to the cone:

FIG. P6.30(b)

$$\sum F_y = ma_y: \quad +n - \frac{mv^2}{r} \sin 49.2^\circ = 0$$

$$n = (2 \times 10^{-3} \text{ kg}) (2.38 \times 10^5 \text{ m/s}^2) \sin 49.2^\circ = \boxed{360 \text{ N}}$$

- (c) $f = \mu_k n = 0.6(360 \text{ N}) = 216 \text{ N}$

$$\sum F_x = ma_x: \quad \frac{mv^2}{r} \cos 49.2^\circ - f = ma_x$$

$$(2 \times 10^{-3} \text{ kg}) (2.38 \times 10^5 \text{ m/s}^2) \cos 49.2^\circ - 216 \text{ N} = (2 \times 10^{-3} \text{ kg}) a_x$$

$$a_x = \boxed{47.5 \times 10^4 \text{ m/s}^2 \text{ radially up the wall of the cone}}$$

P6.31 $a_r = \left(\frac{4\pi^2 R_e}{T^2} \right) \cos 35.0^\circ = 0.0276 \text{ m/s}^2$

We take the y axis along the local vertical.

$$(a_{\text{net}})_y = 9.80 - (a_r)_y = 9.78 \text{ m/s}^2$$

$$(a_{\text{net}})_x = 0.0158 \text{ m/s}^2$$

$$\theta = \arctan \frac{a_x}{a_y} = \boxed{0.0928^\circ}$$

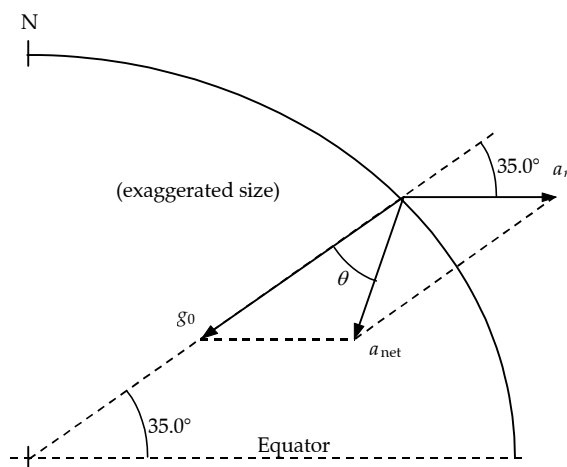


FIG. P6.31

Section 6.4 Motion in the Presence of Resistive Forces

P6.32 $m = 80.0 \text{ kg}$, $v_T = 50.0 \text{ m/s}$, $mg = \frac{D\rho A v_T^2}{2} \therefore \frac{D\rho A}{2} = \frac{mg}{v_T^2} = 0.314 \text{ kg/m}$

(a) At $v = 30.0 \text{ m/s}$

$$a = g - \frac{\frac{D\rho A v^2}{2}}{m} = 9.80 - \frac{(0.314)(30.0)^2}{80.0} = \boxed{6.27 \text{ m/s}^2 \text{ downward}}$$

(b) At $v = 50.0 \text{ m/s}$, terminal velocity has been reached.

$$\sum F_y = 0 = mg - R$$

$$\Rightarrow R = mg = (80.0 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{784 \text{ N directed up}}$$

(c) At $v = 30.0 \text{ m/s}$

$$\frac{D\rho A v^2}{2} = (0.314)(30.0)^2 = \boxed{283 \text{ N}} \text{ upward}$$

P6.33 (a) $a = g - bv$

When $v = v_T$, $a = 0$ and $g = bv_T$ $b = \frac{g}{v_T}$

The Styrofoam falls 1.50 m at constant speed v_T in 5.00 s.

Thus,
$$v_T = \frac{y}{t} = \frac{1.50 \text{ m}}{5.00 \text{ s}} = 0.300 \text{ m/s}$$

Then
$$b = \frac{9.80 \text{ m/s}^2}{0.300 \text{ m/s}} = \boxed{32.7 \text{ s}^{-1}}$$

(b) At $t = 0$, $v = 0$ and $a = g = \boxed{9.80 \text{ m/s}^2}$ down

(c) When $v = 0.150 \text{ m/s}$, $a = g - bv = 9.80 \text{ m/s}^2 - (32.7 \text{ s}^{-1})(0.150 \text{ m/s}) = \boxed{4.90 \text{ m/s}^2}$ down

P6.34 (a) $\rho = \frac{m}{V}$, $A = 0.0201 \text{ m}^2$, $R = \frac{1}{2}\rho_{\text{air}}ADv_T^2 = mg$

$$m = \rho_{\text{bead}}V = 0.830 \text{ g/cm}^3 \left[\frac{4}{3}\pi(8.00 \text{ cm})^3 \right] = 1.78 \text{ kg}$$

Assuming a drag coefficient of $D = 0.500$ for this spherical object, and taking the density of air at 20°C from the endpapers, we have

$$v_T = \sqrt{\frac{2(1.78 \text{ kg})(9.80 \text{ m/s}^2)}{0.500(1.20 \text{ kg/m}^3)(0.0201 \text{ m}^2)}} = \boxed{53.8 \text{ m/s}}$$

(b) $v_f^2 = v_i^2 + 2gh = 0 + 2gh$: $h = \frac{v_f^2}{2g} = \frac{(53.8 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{148 \text{ m}}$

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- P6.35** Since the upward velocity is constant, the resultant force on the ball is zero. Thus, the upward applied force equals the sum of the gravitational and drag forces (both downward):
 $F = mg + bv$.

The mass of the copper ball is

$$m = \frac{4\pi\rho r^3}{3} = \left(\frac{4}{3}\right)\pi(8.92 \times 10^3 \text{ kg/m}^3)(2.00 \times 10^{-2} \text{ m})^3 = 0.299 \text{ kg}.$$

The applied force is then

$$F = mg + bv = (0.299)(9.80) + (0.950)(9.00 \times 10^{-2}) = \boxed{3.01 \text{ N}}.$$

- P6.36** $\sum F_y = ma_y$
 $+T \cos 40.0^\circ - mg = 0$
 $T = \frac{(620 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 40.0^\circ} = 7.93 \times 10^3 \text{ N}$

$$\sum F_x = ma_x$$

$$-R + T \sin 40.0^\circ = 0$$

$$R = (7.93 \times 10^3 \text{ N}) \sin 40.0^\circ = 5.10 \times 10^3 \text{ N} = \frac{1}{2} D \rho A v^2$$

$$D = \frac{2R}{\rho A v^2} = \frac{2(5.10 \times 10^3 \text{ N})\left(\frac{\text{kg m/s}^2}{\text{N}}\right)}{(1.20 \text{ kg/m}^2)(3.80 \text{ m}^2)(40.0 \text{ m/s})^2} = \boxed{1.40}$$

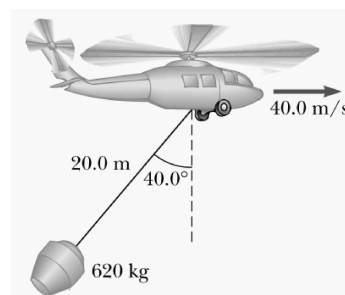


FIG. P6.36

- P6.37** (a) At terminal velocity, $R = v_T b = mg$
 $\therefore b = \frac{mg}{v_T} = \frac{(3.00 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{2.00 \times 10^{-2} \text{ m/s}} = \boxed{1.47 \text{ N} \cdot \text{s/m}}$

- (b) In the equation describing the time variation of the velocity, we have

$$v = v_T (1 - e^{-bt/m}) \quad v = 0.632 v_T \text{ when } e^{-bt/m} = 0.368$$

or at time $t = -\left(\frac{m}{b}\right) \ln(0.368) = \boxed{2.04 \times 10^{-3} \text{ s}}$

- (c) At terminal velocity, $R = v_T b = mg = \boxed{2.94 \times 10^{-2} \text{ N}}$

- P6.38** The resistive force is

$$R = \frac{1}{2} D \rho A v^2 = \frac{1}{2} (0.250)(1.20 \text{ kg/m}^3)(2.20 \text{ m}^2)(27.8 \text{ m/s})^2$$

$$R = 255 \text{ N}$$

$$a = -\frac{R}{m} = -\frac{255 \text{ N}}{1200 \text{ kg}} = \boxed{-0.212 \text{ m/s}^2}$$

P6.39 (a) $v(t) = v_i e^{-ct}$ $v(20.0 \text{ s}) = 5.00 = v_i e^{-20.0c}$, $v_i = 10.0 \text{ m/s}$.

So $5.00 = 10.0 e^{-20.0c}$ and $-20.0c = \ln\left(\frac{1}{2}\right)$ $c = -\frac{\ln\left(\frac{1}{2}\right)}{20.0} = \boxed{3.47 \times 10^{-2} \text{ s}^{-1}}$

(b) At $t = 40.0 \text{ s}$ $v = (10.0 \text{ m/s})e^{-40.0c} = (10.0 \text{ m/s})(0.250) = \boxed{2.50 \text{ m/s}}$

(c) $v = v_i e^{-ct}$ $s = \frac{dv}{dt} = -cv_i e^{-ct} = \boxed{-cv}$

P6.40 $\sum F = ma$

$$-kmv^2 = m \frac{dv}{dt}$$

$$-kdt = \frac{dv}{v^2}$$

$$-k \int_0^t dt = \int_{v_0}^v v^{-2} dv$$

$$-k(t-0) = \frac{v^{-1}}{-1} \Big|_{v_0}^v = -\frac{1}{v} + \frac{1}{v_0}$$

$$\frac{1}{v} = \frac{1}{v_0} + kt = \frac{1 + v_0 kt}{v_0}$$

$$v = \frac{v_0}{1 + v_0 kt}$$

***P6.41** (a) From Problem 40,

$$v = \frac{dx}{dt} = \frac{v_0}{1 + v_0 kt}$$

$$\int_0^x dx = \int_0^t v_0 \frac{dt}{1 + v_0 kt} = \frac{1}{k} \int_0^t \frac{v_0 k dt}{1 + v_0 kt}$$

$$x|_0^x = \frac{1}{k} \ln(1 + v_0 kt) \Big|_0^t$$

$$x - 0 = \frac{1}{k} [\ln(1 + v_0 kt) - \ln 1]$$

$$\boxed{x = \frac{1}{k} \ln(1 + v_0 kt)}$$

(b) We have $\ln(1 + v_0 kt) = kx$

$$1 + v_0 kt = e^{kx} \text{ so } v = \frac{v_0}{1 + v_0 kt} = \frac{v_0}{e^{kx}} = \boxed{v_0 e^{-kx} = v}$$

***P6.42** We write $-kmv^2 = -\frac{1}{2} D \rho A v^2$ so

$$k = \frac{D \rho A}{2m} = \frac{0.305 (1.20 \text{ kg/m}^3) (4.2 \times 10^{-3} \text{ m}^2)}{2(0.145 \text{ kg})} = 5.3 \times 10^{-3} / \text{m}$$

$$v = v_0 e^{-kx} = (40.2 \text{ m/s}) e^{-(5.3 \times 10^{-3} / \text{m})(18.3 \text{ m})} = \boxed{36.5 \text{ m/s}}$$

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P6.43 In $R = \frac{1}{2} D \rho A v^2$, we estimate that $D = 1.00$, $\rho = 1.20 \text{ kg/m}^3$, $A = (0.100 \text{ m})(0.160 \text{ m}) = 1.60 \times 10^{-2} \text{ m}^2$ and $v = 27.0 \text{ m/s}$. The resistance force is then

$$R = \frac{1}{2} (1.00) (1.20 \text{ kg/m}^3) (1.60 \times 10^{-2} \text{ m}^2) (27.0 \text{ m/s})^2 = 7.00 \text{ N}$$

or

$$R \sim \boxed{10^1 \text{ N}}$$

Section 6.5 Numerical Modeling in Particle Dynamics

Note: In some problems we compute each new position as $x(t + \Delta t) = x(t) + v(t + \Delta t) \Delta t$, rather than $x(t + \Delta t) = x(t) + v(t) \Delta t$ as quoted in the text. This method has the same theoretical validity as that presented in the text, and in practice can give quicker convergence.

P6.44 (a) At $v = v_T$, $a = 0$, $-mg + bv_T = 0$ $v_T = \frac{mg}{b} = \frac{(3.00 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{3.00 \times 10^{-2} \text{ kg/s}} = \boxed{0.980 \text{ m/s}}$

(b)

$t(\text{s})$	$x(\text{m})$	$v(\text{m/s})$	$F(\text{mN})$	$a(\text{m/s}^2)$
0	2	0	-29.4	-9.8
0.005	2	-0.049	-27.93	-9.31
0.01	1.999 755	-0.095 55	-26.534	-8.844 5
0.015	1.999 3	-0.139 77	-25.2	-8.40

... we list the result after each tenth iteration

0.05	1.990	-0.393	-17.6	-5.87
0.1	1.965	-0.629	-10.5	-3.51
0.15	1.930	-0.770	-6.31	-2.10
0.2	1.889	-0.854	-3.78	-1.26
0.25	1.845	-0.904	-2.26	-0.754
0.3	1.799	-0.935	-1.35	-0.451
0.35	1.752	-0.953	-0.811	-0.270
0.4	1.704	-0.964	-0.486	-0.162
0.45	1.65	-0.970	-0.291	-0.096 9
0.5	1.61	-0.974	-0.174	-0.058 0
0.55	1.56	-0.977	-0.110	-0.034 7
0.6	1.51	-0.978	-0.062 4	-0.020 8
0.65	1.46	-0.979	-0.037 4	-0.012 5

Terminal velocity is never reached. The leaf is at 99.9% of v_T after 0.67 s. The fall to the ground takes about 2.14 s. Repeating with $\Delta t = 0.001 \text{ s}$, we find the fall takes 2.14 s.

- P6.45** (a) When $v = v_T$, $a = 0$, $\sum F = -mg + Cv_T^2 = 0$

$$v_T = -\sqrt{\frac{mg}{C}} = -\sqrt{\frac{(4.80 \times 10^{-4} \text{ kg})(9.80 \text{ m/s}^2)}{2.50 \times 10^{-5} \text{ kg/m}}} = \boxed{-13.7 \text{ m/s}}$$

(b)

$t(\text{s})$	$x(\text{m})$	$v(\text{m/s})$	$F(\text{mN})$	$a(\text{m/s}^2)$
0	0	0	-4.704	-9.8
0.2	0	-1.96	-4.608	-9.599 9
0.4	-0.392	-3.88	-4.327 6	-9.015 9
0.6	-1.168	-5.683 2	-3.896 5	-8.117 8
0.8	-2.30	-7.306 8	-3.369 3	-7.019 3
1.0	-3.77	-8.710 7	-2.807 1	-5.848 1
1.2	-5.51	-9.880 3	-2.263 5	-4.715 6
1.4	-7.48	-10.823	-1.775 3	-3.698 6
1.6	-9.65	-11.563	-1.361 6	-2.836 6
1.8	-11.96	-12.13	-1.03	-2.14
2	-14.4	-12.56	-0.762	-1.59

... listing results after each fifth step

3	-27.4	-13.49	-0.154	-0.321
4	-41.0	-13.67	-0.029 1	-0.060 6
5	-54.7	-13.71	-0.005 42	-0.011 3

The hailstone reaches 99% of v_T after 3.3 s, 99.95% of v_T after 5.0 s, 99.99% of v_T after 6.0 s, 99.999% of v_T after 7.4 s.

- P6.46** (a) At terminal velocity, $\sum F = 0 = -mg + Cv_T^2$

$$C = \frac{mg}{v_T^2} = \frac{(0.142 \text{ kg})(9.80 \text{ m/s}^2)}{(42.5 \text{ m/s})^2} = \boxed{7.70 \times 10^{-4} \text{ kg/m}}$$

(b) $Cv^2 = (7.70 \times 10^{-4} \text{ kg/m})(36.0 \text{ m/s})^2 = \boxed{0.998 \text{ N}}$

(c)

Elapsed Time (s)	Altitude (m)	Speed (m/s)	Resistance Force (N)	Net Force (N)	Acceleration (m/s^2)
0.000 00	0.000 00	36.000 00	-0.998 49	-2.390 09	-16.831 58
0.050 00	1.757 92	35.158 42	-0.952 35	-2.343 95	-16.506 67
...					
2.950 00	48.623 27	0.824 94	-0.000 52	-1.392 12	-9.803 69
3.000 00	48.640 00	0.334 76	-0.000 09	-1.391 69	-9.800 61
3.050 00	48.632 24	-0.155 27	0.000 02	-1.391 58	-9.799 87
...					
6.250 00	1.250 85	-26.852 97	0.555 55	-0.836 05	-5.887 69
6.300 00	-0.106 52	-27.147 36	0.567 80	-0.823 80	-5.801 44

Maximum height is about $\boxed{49 \text{ m}}$. It returns to the ground after about $\boxed{6.3 \text{ s}}$ with a speed of approximately $\boxed{27 \text{ m/s}}$.

P6.47 (a) At constant velocity $\sum F = 0 = -mg + Cv_T^2$

$$v_T = -\sqrt{\frac{mg}{C}} = -\sqrt{\frac{(50.0 \text{ kg})(9.80 \text{ m/s}^2)}{0.200 \text{ kg/m}}} = \boxed{-49.5 \text{ m/s}} \text{ with chute closed and}$$

$$v_T = -\sqrt{\frac{(50.0 \text{ kg})(9.80 \text{ m/s})}{20.0 \text{ kg/m}}} = \boxed{-4.95 \text{ m/s}} \text{ with chute open.}$$

- (b) We use time increments of 0.1 s for $0 < t < 10 \text{ s}$, then 0.01 s for $10 \text{ s} < t < 12 \text{ s}$, and then 0.1 s again.

time(s)	height(m)	velocity(m/s)
0	1000	0
1	995	-9.7
2	980	-18.6
4	929	-32.7
7	812	-43.7
10	674	-47.7
10.1	671	-16.7
10.3	669	-8.02
11	665	-5.09
12	659	-4.95
50	471	-4.95
100	224	-4.95
145	0	-4.95

- 6.48 (a) We use a time increment of 0.01 s.

time(s)	x(m)	y(m)
0	0	0
0.100	7.81	5.43
0.200	14.9	10.2
0.400	27.1	18.3
1.00	51.9	32.7
1.92	70.0	38.5
2.00	70.9	38.5
4.00	80.4	26.7
5.00	81.4	17.7
6.85	81.8	0

with θ	we find range
30.0°	86.410 m
35.0°	81.8 m
25.0°	90.181 m
20.0°	92.874 m
15.0°	93.812 m
10.0°	90.965 m
17.0°	93.732 m
16.0°	93.839 8 m
15.5°	93.829 m
15.8°	93.839 m
16.1°	93.838 m
15.9°	93.840 2 m

(b) range = $\boxed{81.8 \text{ m}}$

(c) So we have maximum range at $\theta = \boxed{15.9^\circ}$

- P6.49** (a) At terminal speed, $\sum F = -mg + Cv^2 = 0$. Thus,

$$C = \frac{mg}{v^2} = \frac{(0.0460 \text{ kg})(9.80 \text{ m/s}^2)}{(44.0 \text{ m/s})^2} = \boxed{2.33 \times 10^{-4} \text{ kg/m}}$$

- (b) We set up a spreadsheet to calculate the motion, try different initial speeds, and home in on $\boxed{53 \text{ m/s}}$ as that required for horizontal range of 155 m, thus:

Time t (s)	x (m)	v_x (m/s)	a_x (m/s ²)	y (m)	v_y (m/s)	a_y (m/s ²)	$v = \sqrt{v_x^2 + v_y^2}$ (m/s)	$\tan^{-1}\left(\frac{v_y}{v_x}\right)$ (deg)
0.000 0	0.000 0	45.687 0	-10.565 9	0.000 0	27.451 5	-13.614 6	53.300 0	31.000 0
0.002 7	0.121 1	45.659 0	-10.552 9	0.072 7	27.415 5	-13.604 6	53.257 4	30.982 2
...								
2.501 6	90.194 6	28.937 5	-4.238 8	32.502 4	0.023 5	-9.800 0	28.937 5	0.046 6
2.504 3	90.271 3	28.926 3	-4.235 5	32.502 4	-0.002 4	-9.800 0	28.926 3	-0.004 8
2.506 9	90.348 0	28.915 0	-4.232 2	32.502 4	-0.028 4	-9.800 0	28.915 1	-0.056 3
...								
3.423 8	115.229 8	25.492 6	-3.289 6	28.397 2	-8.890 5	-9.399 9	26.998 4	-19.226 2
3.426 5	115.297 4	25.483 9	-3.287 4	28.373 6	-8.915 4	-9.397 7	26.998 4	-19.282 2
3.429 1	115.364 9	25.475 1	-3.285 1	28.350 0	-8.940 3	-9.395 4	26.998 4	-19.338 2
...								
5.151 6	154.996 8	20.843 8	-2.199 2	0.005 9	-23.308 7	-7.049 8	31.269 2	-48.195 4
5.154 3	155.052 0	20.838 0	-2.198 0	-0.055 9	-23.327 4	-7.045 4	31.279 2	-48.226 2

- (c) Similarly, the initial speed is $\boxed{42 \text{ m/s}}$. The motion proceeds thus:

Time t (s)	x (m)	v_x (m/s)	a_x (m/s ²)	y (m)	v_y (m/s)	a_y (m/s ²)	$v = \sqrt{v_x^2 + v_y^2}$ (m/s)	$\tan^{-1}\left(\frac{v_y}{v_x}\right)$ (deg)
0.000 0	0.000 0	28.746 2	-4.182 9	0.000 0	30.826 6	-14.610 3	42.150 0	47.000 0
0.003 5	0.100 6	28.731 6	-4.178 7	0.107 9	30.775 4	-14.594 3	42.102 6	46.967 1
...								
2.740 5	66.307 8	20.548 4	-2.137 4	39.485 4	0.026 0	-9.800 0	20.548 5	0.072 5
2.744 0	66.379 7	20.541 0	-2.135 8	39.485 5	-0.008 3	-9.800 0	20.541 0	-0.023 1
2.747 5	66.451 6	20.533 5	-2.134 3	39.485 5	-0.042 6	-9.800 0	20.533 5	-0.118 8
...								
3.146 5	74.480 5	19.715 6	-1.967 6	38.696 3	-3.942 3	-9.721 3	20.105 8	-11.307 7
3.150 0	74.549 5	19.708 7	-1.966 2	38.682 5	-3.976 4	-9.720 0	20.105 8	-11.406 7
3.153 5	74.618 5	19.701 8	-1.964 9	38.668 6	-4.010 4	-9.718 6	20.105 8	-11.505 6
...								
5.677 0	118.969 7	15.739 4	-1.254 0	0.046 5	-25.260 0	-6.570 1	29.762 3	-58.073 1
5.680 5	119.024 8	15.735 0	-1.253 3	-0.041 9	-25.283 0	-6.564 2	29.779 5	-58.103 7

The trajectory in (c) reaches maximum height 39 m, as opposed to 33 m in (b). In both, the ball reaches maximum height when it has covered about 57% of its range. Its speed is a minimum somewhat later. The impact speeds are both about 30 m/s.

Additional Problems

- *P6.50 When the cloth is at a lower angle θ , the radial component of $\sum F = ma$ reads

$$n + mg \sin \theta = \frac{mv^2}{r}.$$

At $\theta = 68.0^\circ$, the normal force drops to zero and

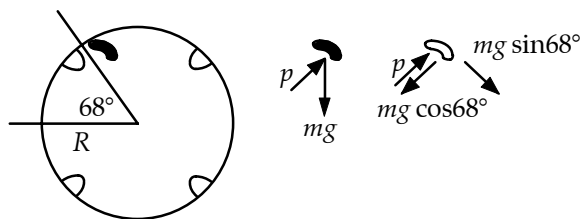
$$g \sin 68^\circ = \frac{v^2}{r}.$$


FIG. P6.50

$$v = \sqrt{rg \sin 68^\circ} = \sqrt{(0.33 \text{ m})(9.8 \text{ m/s}^2) \sin 68^\circ} = 1.73 \text{ m/s}$$

The rate of revolution is

$$\text{angular speed} = (1.73 \text{ m/s}) \left(\frac{1 \text{ rev}}{2\pi r} \right) \left(\frac{2\pi r}{2\pi(0.33 \text{ m})} \right) = \boxed{0.835 \text{ rev/s}} = 50.1 \text{ rev/min}.$$

- *P6.51 (a) $v = (30 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) = 8.33 \text{ m/s}$

$$\begin{aligned} \sum F_y = ma_y: +n - mg &= -\frac{mv^2}{r} \\ n &= m \left(g - \frac{v^2}{r} \right) = 1800 \text{ kg} \left[9.8 \text{ m/s}^2 - \frac{(8.33 \text{ m/s})^2}{20.4 \text{ m}} \right] \\ &= \boxed{1.15 \times 10^4 \text{ N up}} \end{aligned}$$

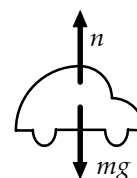


FIG. P6.51

- (b) Take $n = 0$. Then $mg = \frac{mv^2}{r}$.

$$v = \sqrt{gr} = \sqrt{(9.8 \text{ m/s}^2)(20.4 \text{ m})} = \boxed{14.1 \text{ m/s}} = 50.9 \text{ km/h}$$

- P6.52 (a) $\sum F_y = ma_y = \frac{mv^2}{R}$

$$mg - n = \frac{mv^2}{R} \quad n = \boxed{mg - \frac{mv^2}{R}}$$

- (b) When $n = 0$,
- $$mg = \frac{mv^2}{R}$$

Then,

$$v = \boxed{\sqrt{gR}}.$$

- *P6.53** (a) $\text{slope} = \frac{0.160 \text{ N} - 0}{9.9 \text{ m}^2/\text{s}^2} = \boxed{0.0162 \text{ kg/m}}$
- (b) $\text{slope} = \frac{R}{v^2} = \frac{\frac{1}{2}D\rho Av^2}{v^2} = \boxed{\frac{1}{2}D\rho A}$
- (c) $\frac{1}{2}D\rho A = 0.0162 \text{ kg/m}$
 $D = \frac{2(0.0162 \text{ kg/m})}{(1.20 \text{ kg/m}^3)\pi(0.105 \text{ m})^2} = \boxed{0.778}$
- (d) From the table, the eighth point is at force $mg = 8(1.64 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2) = 0.129 \text{ N}$ and horizontal coordinate $(2.80 \text{ m/s})^2$. The vertical coordinate of the line is here $(0.0162 \text{ kg/m})(2.8 \text{ m/s})^2 = 0.127 \text{ N}$. The scatter percentage is $\frac{0.129 \text{ N} - 0.127 \text{ N}}{0.127 \text{ N}} = 1.5\%$.
- (e) The interpretation of the graph can be stated thus: For stacked coffee filters falling at terminal speed, a graph of air resistance force as a function of squared speed demonstrates that the force is proportional to the speed squared within the experimental uncertainty estimated as 2%. This proportionality agrees with that described by the theoretical equation $R = \frac{1}{2}D\rho Av^2$. The value of the constant slope of the graph implies that the drag coefficient for coffee filters is $D = 0.78 \pm 2\%$.

- P6.54** (a) While the car negotiates the curve, the accelerometer is at the angle θ .

Horizontally: $T \sin \theta = \frac{mv^2}{r}$

Vertically: $T \cos \theta = mg$

where r is the radius of the curve, and v is the speed of the car.

By division, $\tan \theta = \frac{v^2}{rg}$

Then $a_c = \frac{v^2}{r} = g \tan \theta$:

$$a_c = (9.80 \text{ m/s}^2) \tan 15.0^\circ$$

$$a_c = \boxed{2.63 \text{ m/s}^2}$$

(b) $r = \frac{v^2}{a_c} \quad r = \frac{(23.0 \text{ m/s})^2}{2.63 \text{ m/s}^2} = \boxed{201 \text{ m}}$

(c) $v^2 = rg \tan \theta = (201 \text{ m})(9.80 \text{ m/s}^2) \tan 9.00^\circ \quad v = \boxed{17.7 \text{ m/s}}$

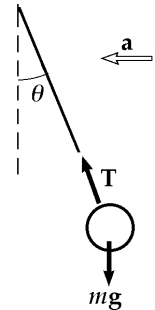


FIG. P6.54

P6.55 Take x -axis up the hill

$$\begin{aligned}\sum F_x = ma_x: & +T \sin \theta - mg \sin \phi = ma \\ & a = \frac{T}{m} \sin \theta - g \sin \phi \\ \sum F_y = ma_y: & +T \cos \theta - mg \cos \phi = 0 \\ & T = \frac{mg \cos \phi}{\cos \theta} \\ & a = \frac{g \cos \phi \sin \theta}{\cos \theta} - g \sin \phi \\ & a = \boxed{g(\cos \phi \tan \theta - \sin \phi)}\end{aligned}$$

- *P6.56 (a) The speed of the bag is $\frac{2\pi(7.46 \text{ m})}{38 \text{ s}} = 1.23 \text{ m/s}$. The total force on it must add to

$$ma_c = \frac{(30 \text{ kg})(1.23 \text{ m/s})^2}{7.46 \text{ m}} = 6.12 \text{ N}$$

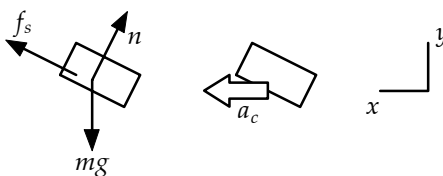


FIG. P6.56

$$\begin{aligned}\sum F_x = ma_x: & f_s \cos 20^\circ - n \sin 20^\circ = 6.12 \text{ N} \\ \sum F_y = ma_y: & f_s \sin 20^\circ + n \cos 20^\circ - (30 \text{ kg})(9.8 \text{ m/s}^2) = 0 \\ & n = \frac{f_s \cos 20^\circ - 6.12 \text{ N}}{\sin 20^\circ}\end{aligned}$$

Substitute:

$$\begin{aligned}f_s \sin 20^\circ + f_s \frac{\cos^2 20^\circ}{\sin 20^\circ} - (6.12 \text{ N}) \frac{\cos 20^\circ}{\sin 20^\circ} &= 294 \text{ N} \\ f_s (2.92) &= 294 \text{ N} + 16.8 \text{ N} \\ f_s &= \boxed{106 \text{ N}}\end{aligned}$$

$$\begin{aligned}\text{(b)} \quad v &= \frac{2\pi(7.94 \text{ m})}{34 \text{ s}} = 1.47 \text{ m/s} \\ ma_c &= \frac{(30 \text{ kg})(1.47 \text{ m/s})^2}{7.94 \text{ m}} = 8.13 \text{ N} \\ f_s \cos 20^\circ - n \sin 20^\circ &= 8.13 \text{ N} \\ f_s \sin 20^\circ + n \cos 20^\circ &= 294 \text{ N} \\ n &= \frac{f_s \cos 20^\circ - 8.13 \text{ N}}{\sin 20^\circ} \\ f_s \sin 20^\circ + f_s \frac{\cos^2 20^\circ}{\sin 20^\circ} - (8.13 \text{ N}) \frac{\cos 20^\circ}{\sin 20^\circ} &= 294 \text{ N} \\ f_s (2.92) &= 294 \text{ N} + 22.4 \text{ N} \\ f_s &= 108 \text{ N} \\ n &= \frac{(108 \text{ N}) \cos 20^\circ - 8.13 \text{ N}}{\sin 20^\circ} = 273 \text{ N} \\ \mu_s = \frac{f_s}{n} &= \frac{108 \text{ N}}{273 \text{ N}} = \boxed{0.396}\end{aligned}$$

- P6.57** (a) Since the centripetal acceleration of a person is downward (toward the axis of the earth), it is equivalent to the effect of a falling elevator. Therefore,

$$F'_g = F_g - \frac{mv^2}{r} \text{ or } \boxed{F_g > F'_g}$$

- (b) At the poles $v = 0$ and $F'_g = F_g = mg = 75.0(9.80) = \boxed{735 \text{ N}}$ down.

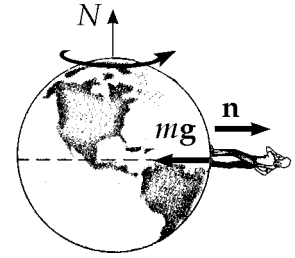


FIG. P6.57

At the equator, $F'_g = F_g - ma_c = 735 \text{ N} - 75.0(0.0337) \text{ N} = \boxed{732 \text{ N}}$ down.

- P6.58** (a) Since the object of mass m_2 is in equilibrium, $\sum F_y = T - m_2g = 0$
or $T = \boxed{m_2g}$.

- (b) The tension in the string provides the required centripetal acceleration of the puck.

Thus,

$$F_c = T = \boxed{m_2g}.$$

- (c) From

$$F_c = \frac{m_1 v^2}{R}$$

we have

$$v = \sqrt{\frac{RF_c}{m_1}} = \boxed{\sqrt{\left(\frac{m_2}{m_1}\right)gR}}.$$

- P6.59** (a) $v = (300 \text{ mi/h}) \left(\frac{88.0 \text{ ft/s}}{60.0 \text{ mi/h}} \right) = 440 \text{ ft/s}$

At the lowest point, his seat exerts an upward force; therefore, his weight seems to increase. His apparent weight is

$$F'_g = mg + m \frac{v^2}{r} = 160 + \left(\frac{160}{32.0} \right) \frac{(440)^2}{1200} = \boxed{967 \text{ lb}}.$$

- (b) At the highest point, the force of the seat on the pilot is directed down and

$$F'_g = mg - m \frac{v^2}{r} = \boxed{-647 \text{ lb}}.$$

Since the plane is upside down, the seat exerts this downward force.

- (c) When $F'_g = 0$, then $mg = \frac{mv^2}{R}$. If we vary the aircraft's R and v such that the above is true, then the pilot feels weightless.

P6.60 For the block to remain stationary, $\sum F_y = 0$ and $\sum F_x = ma_r$.

$$n_1 = (m_p + m_b)g \text{ so } f \leq \mu_{s1}n_1 = \mu_{s1}(m_p + m_b)g.$$

At the point of slipping, the required centripetal force equals the maximum friction force:

$$\therefore (m_p + m_b) \frac{v_{\max}^2}{r} = \mu_{s1}(m_p + m_b)g$$

$$\text{or } v_{\max} = \sqrt{\mu_{s1}rg} = \sqrt{(0.750)(0.120)(9.80)} = 0.939 \text{ m/s}.$$

For the penny to remain stationary on the block:

$$\sum F_y = 0 \Rightarrow n_2 - m_p g = 0 \text{ or } n_2 = m_p g$$

$$\text{and } \sum F_x = ma_r \Rightarrow f_p = m_p \frac{v^2}{r}.$$

When the penny is about to slip on the block, $f_p = f_{p, \max} = \mu_{s2}n_2$

$$\text{or } \mu_{s2}m_p g = m_p \frac{v_{\max}^2}{r}$$

$$v_{\max} = \sqrt{\mu_{s2}rg} = \sqrt{(0.520)(0.120)(9.80)} = 0.782 \text{ m/s}$$

This is less than the maximum speed for the block, so the penny slips before the block starts to slip. The maximum rotation frequency is

$$\text{Max rpm} = \frac{v_{\max}}{2\pi r} = (0.782 \text{ m/s}) \left[\frac{1 \text{ rev}}{2\pi(0.120 \text{ m})} \right] \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = \boxed{62.2 \text{ rev/min}}.$$

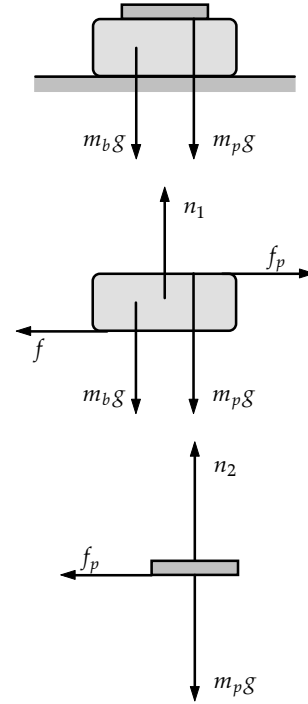


FIG. P6.60

P6.61 $v = \frac{2\pi r}{T} = \frac{2\pi(9.00 \text{ m})}{(15.0 \text{ s})} = 3.77 \text{ m/s}$

(a) $a_r = \frac{v^2}{r} = \boxed{1.58 \text{ m/s}^2}$

(b) $F_{\text{low}} = m(g + a_r) = \boxed{455 \text{ N}}$

(c) $F_{\text{high}} = m(g - a_r) = \boxed{328 \text{ N}}$

(d) $F_{\text{mid}} = m\sqrt{g^2 + a_r^2} = \boxed{397 \text{ N upward and}} \text{ at } \theta = \tan^{-1} \frac{a_r}{g} = \tan^{-1} \frac{1.58}{9.8} = \boxed{9.15^\circ \text{ inward}}.$

P6.62 Standing on the inner surface of the rim, and moving with it, each person will feel a normal force exerted by the rim. This inward force causes the 3.00 m/s^2 centripetal acceleration:

$$a_c = \frac{v^2}{r}:$$

$$v = \sqrt{a_c r} = \sqrt{(3.00 \text{ m/s}^2)(60.0 \text{ m})} = 13.4 \text{ m/s}$$

$$\text{The period of rotation comes from } v = \frac{2\pi r}{T}: T = \frac{2\pi r}{v} = \frac{2\pi(60.0 \text{ m})}{13.4 \text{ m/s}} = 28.1 \text{ s}$$

so the frequency of rotation is

$$f = \frac{1}{T} = \frac{1}{28.1 \text{ s}} = \frac{1}{28.1 \text{ s}} \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = \boxed{2.14 \text{ rev/min}}.$$

- P6.63** (a) The mass at the end of the chain is in vertical equilibrium. Thus $T \cos \theta = mg$.

$$\text{Horizontally } T \sin \theta = ma_r = \frac{mv^2}{r}$$

$$r = (2.50 \sin \theta + 4.00) \text{ m}$$

$$r = (2.50 \sin 28.0^\circ + 4.00) \text{ m} = 5.17 \text{ m}$$

$$\text{Then } a_r = \frac{v^2}{5.17 \text{ m}}.$$

$$\text{By division } \tan \theta = \frac{a_r}{g} = \frac{v^2}{5.17g}$$

$$v^2 = 5.17g \tan \theta = (5.17)(9.80)(\tan 28.0^\circ) \text{ m}^2/\text{s}^2$$

$$v = \boxed{5.19 \text{ m/s}}$$

- (b) $T \cos \theta = mg$

$$T = \frac{mg}{\cos \theta} = \frac{(50.0 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 28.0^\circ} = \boxed{555 \text{ N}}$$

- P6.64** (a) The putty, when dislodged, rises and returns to the original level in time t . To find t , we use $v_f = v_i + at$: i.e., $-v = +v - gt$ or $t = \frac{2v}{g}$ where v is the speed of a point on the rim of the wheel.

$$\text{If } R \text{ is the radius of the wheel, } v = \frac{2\pi R}{t}, \text{ so } t = \frac{2v}{g} = \frac{2\pi R}{v}.$$

$$\text{Thus, } v^2 = \pi Rg \text{ and } \boxed{v = \sqrt{\pi Rg}}.$$

- (b) The putty is dislodged when F , the force holding it to the wheel is

$$F = \frac{mv^2}{R} = \boxed{m\pi g}.$$

- P6.65** (a) $n = \frac{mv^2}{R}$ $f - mg = 0$

$$f = \mu_s n \quad v = \frac{2\pi R}{T}$$

$$T = \sqrt{\frac{4\pi^2 R \mu_s}{g}}$$

- (b) $T = \boxed{2.54 \text{ s}}$

$$\# \frac{\text{rev}}{\text{min}} = \frac{1 \text{ rev}}{2.54 \text{ s}} \left(\frac{60 \text{ s}}{\text{min}} \right) = \boxed{23.6 \frac{\text{rev}}{\text{min}}}$$

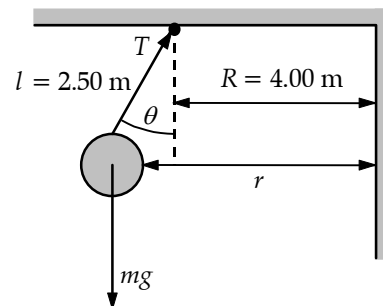


FIG. P6.63

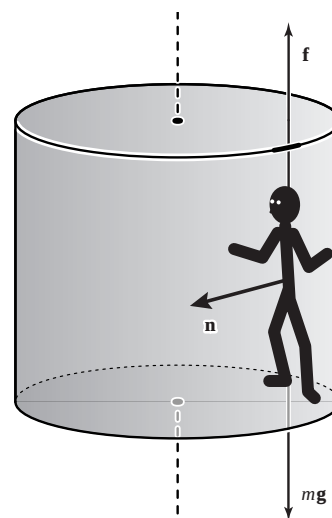


FIG. P6.65

P6.66 Let the x -axis point eastward, the y -axis upward, and the z -axis point southward.

(a) The range is $Z = \frac{v_i^2 \sin 2\theta_i}{g}$

The initial speed of the ball is therefore

$$v_i = \sqrt{\frac{gZ}{\sin 2\theta_i}} = \sqrt{\frac{(9.80)(285)}{\sin 96.0^\circ}} = 53.0 \text{ m/s}$$

The time the ball is in the air is found from $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ as

$$0 = (53.0 \text{ m/s})(\sin 48.0^\circ)t - (4.90 \text{ m/s}^2)t^2$$

giving $t = \boxed{8.04 \text{ s}}$.

(b) $v_{ix} = \frac{2\pi R_e \cos \phi_i}{86\,400 \text{ s}} = \frac{2\pi(6.37 \times 10^6 \text{ m}) \cos 35.0^\circ}{86\,400 \text{ s}} = \boxed{379 \text{ m/s}}$

(c) 360° of latitude corresponds to a distance of $2\pi R_e$, so 285 m is a change in latitude of

$$\Delta\phi = \left(\frac{S}{2\pi R_e}\right)(360^\circ) = \left(\frac{285 \text{ m}}{2\pi(6.37 \times 10^6 \text{ m})}\right)(360^\circ) = 2.56 \times 10^{-3} \text{ degrees}$$

The final latitude is then $\phi_f = \phi_i - \Delta\phi = 35.0^\circ - 0.002\,56^\circ = 34.997\,4^\circ$.

The cup is moving eastward at a speed $v_{fx} = \frac{2\pi R_e \cos \phi_f}{86\,400 \text{ s}}$, which is larger than the eastward velocity of the tee by

$$\begin{aligned} \Delta v_x &= v_{fx} - v_{fi} = \frac{2\pi R_e}{86\,400 \text{ s}} [\cos \phi_f - \cos \phi_i] = \frac{2\pi R_e}{86\,400 \text{ s}} [\cos(\phi_i - \Delta\phi) - \cos \phi_i] \\ &= \frac{2\pi R_e}{86\,400 \text{ s}} [\cos \phi_i \cos \Delta\phi + \sin \phi_i \sin \Delta\phi - \cos \phi_i] \end{aligned}$$

Since $\Delta\phi$ is such a small angle, $\cos \Delta\phi \approx 1$ and $\Delta v_x \approx \frac{2\pi R_e}{86\,400 \text{ s}} \sin \phi_i \sin \Delta\phi$.

$$\Delta v_x \approx \frac{2\pi(6.37 \times 10^6 \text{ m})}{86\,400 \text{ s}} \sin 35.0^\circ \sin 0.002\,56^\circ = \boxed{1.19 \times 10^{-2} \text{ m/s}}$$

(d) $\Delta x = (\Delta v_x)t = (1.19 \times 10^{-2} \text{ m/s})(8.04 \text{ s}) = 0.095\,5 \text{ m} = \boxed{9.55 \text{ cm}}$

- P6.67** (a) If the car is about to slip *down* the incline, f is directed up the incline.

$\sum F_y = n \cos \theta + f \sin \theta - mg = 0$ where $f = \mu_s n$ gives

$$n = \frac{mg}{\cos \theta (1 + \mu_s \tan \theta)} \text{ and } f = \frac{\mu_s mg}{\cos \theta (1 + \mu_s \tan \theta)}.$$

Then, $\sum F_x = n \sin \theta - f \cos \theta = m \frac{v_{\min}^2}{R}$ yields

$$v_{\min} = \sqrt{\frac{Rg(\tan \theta - \mu_s)}{1 + \mu_s \tan \theta}}.$$

When the car is about to slip *up* the incline, f is directed down the incline. Then, $\sum F_y = n \cos \theta - f \sin \theta - mg = 0$ with $f = \mu_s n$ yields

$$n = \frac{mg}{\cos \theta (1 - \mu_s \tan \theta)} \text{ and } f = \frac{\mu_s mg}{\cos \theta (1 - \mu_s \tan \theta)}.$$

In this case, $\sum F_x = n \sin \theta + f \cos \theta = m \frac{v_{\max}^2}{R}$, which gives

$$v_{\max} = \sqrt{\frac{Rg(\tan \theta + \mu_s)}{1 - \mu_s \tan \theta}}.$$

- (b) If $v_{\min} = \sqrt{\frac{Rg(\tan \theta - \mu_s)}{1 + \mu_s \tan \theta}} = 0$, then $\mu_s = \tan \theta$.

(c)
$$v_{\min} = \sqrt{\frac{(100 \text{ m})(9.80 \text{ m/s}^2)(\tan 10.0^\circ - 0.100)}{1 + (0.100) \tan 10.0^\circ}} = 8.57 \text{ m/s}$$

$$v_{\max} = \sqrt{\frac{(100 \text{ m})(9.80 \text{ m/s}^2)(\tan 10.0^\circ + 0.100)}{1 - (0.100) \tan 10.0^\circ}} = 16.6 \text{ m/s}$$

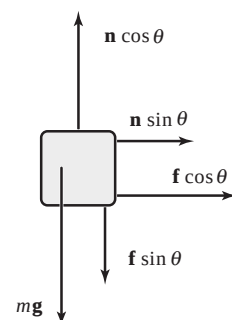
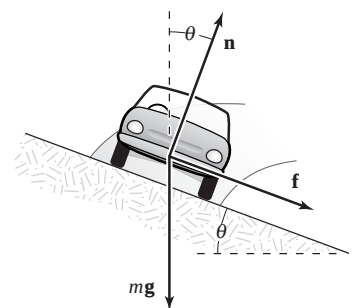
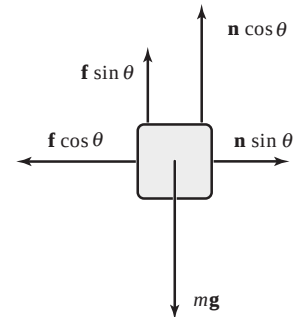
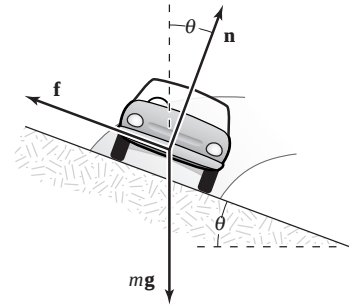


FIG. P6.67

- P6.68 (a) The bead moves in a circle with radius $v = R \sin \theta$ at a speed of

$$v = \frac{2\pi r}{T} = \frac{2\pi R \sin \theta}{T}$$

The normal force has
an inward radial component of $n \sin \theta$
and an upward component of $n \cos \theta$

$$\sum F_y = ma_y: n \cos \theta - mg = 0$$

or

$$n = \frac{mg}{\cos \theta}$$

Then $\sum F_x = n \sin \theta = m \frac{v^2}{r}$ becomes $\left(\frac{mg}{\cos \theta} \right) \sin \theta = \frac{m}{R \sin \theta} \left(\frac{2\pi R \sin \theta}{T} \right)^2$

which reduces to $\frac{g \sin \theta}{\cos \theta} = \frac{4\pi^2 R \sin \theta}{T^2}$

This has two solutions: $\sin \theta = 0 \Rightarrow \theta = 0^\circ$ (1)

and $\cos \theta = \frac{gT^2}{4\pi^2 R}$ (2)

If $R = 15.0$ cm and $T = 0.450$ s, the second solution yields

$$\cos \theta = \frac{(9.80 \text{ m/s}^2)(0.450 \text{ s})^2}{4\pi^2(0.150 \text{ m})} = 0.335 \text{ and } \theta = 70.4^\circ$$

Thus, in this case, the bead can ride at two positions $\theta = 70.4^\circ$ and $\theta = 0^\circ$.

- (b) At this slower rotation, solution (2) above becomes

$$\cos \theta = \frac{(9.80 \text{ m/s}^2)(0.850 \text{ s})^2}{4\pi^2(0.150 \text{ m})} = 1.20, \text{ which is impossible.}$$

In this case, the bead can ride only at the bottom of the loop, $\theta = 0^\circ$. The loop's rotation must be faster than a certain threshold value in order for the bead to move away from the lowest position.

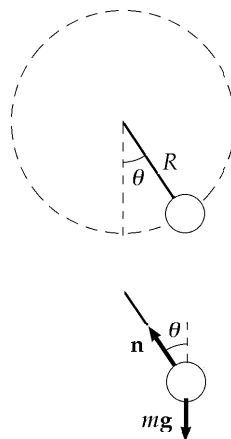


FIG. P6.68(a)

P6.69 At terminal velocity, the accelerating force of gravity is balanced by frictional drag: $mg = arv + br^2v^2$

$$(a) \quad mg = (3.10 \times 10^{-9})v + (0.870 \times 10^{-10})v^2$$

$$\text{For water,} \quad m = \rho V = 1000 \text{ kg/m}^3 \left[\frac{4}{3} \pi (10^{-5} \text{ m})^3 \right]$$

$$4.11 \times 10^{-11} = (3.10 \times 10^{-9})v + (0.870 \times 10^{-10})v^2$$

Assuming v is small, ignore the second term on the right hand side: $v = 0.0132 \text{ m/s}$.

$$(b) \quad mg = (3.10 \times 10^{-8})v + (0.870 \times 10^{-8})v^2$$

Here we cannot ignore the second term because the coefficients are of nearly equal magnitude.

$$4.11 \times 10^{-8} = (3.10 \times 10^{-8})v + (0.870 \times 10^{-8})v^2$$

$$v = \frac{-3.10 \pm \sqrt{(3.10)^2 + 4(0.870)(4.11)}}{2(0.870)} = 1.03 \text{ m/s}$$

$$(c) \quad mg = (3.10 \times 10^{-7})v + (0.870 \times 10^{-6})v^2$$

Assuming $v > 1 \text{ m/s}$, and ignoring the first term:

$$4.11 \times 10^{-5} = (0.870 \times 10^{-6})v^2 \quad v = 6.87 \text{ m/s}$$

P6.70 $v = \left(\frac{mg}{b} \right) \left[1 - \exp\left(\frac{-bt}{m} \right) \right]$ where $\exp(x) = e^x$ is the exponential function.

$$\text{At } t \rightarrow \infty, \quad v \rightarrow v_T = \frac{mg}{b}$$

$$\text{At } t = 5.54 \text{ s} \quad 0.500v_T = v_T \left[1 - \exp\left(\frac{-b(5.54 \text{ s})}{9.00 \text{ kg}} \right) \right]$$

$$\exp\left(\frac{-b(5.54 \text{ s})}{9.00 \text{ kg}} \right) = 0.500;$$

$$\frac{-b(5.54 \text{ s})}{9.00 \text{ kg}} = \ln 0.500 = -0.693;$$

$$b = \frac{(9.00 \text{ kg})(0.693)}{5.54 \text{ s}} = 1.13 \text{ m/s}$$

$$(a) \quad v_T = \frac{mg}{b} \quad v_T = \frac{(9.00 \text{ kg})(9.80 \text{ m/s}^2)}{1.13 \text{ kg/s}} = 78.3 \text{ m/s}$$

$$(b) \quad 0.750v_T = v_T \left[1 - \exp\left(\frac{-1.13t}{9.00 \text{ s}} \right) \right] \quad \exp\left(\frac{-1.13t}{9.00 \text{ s}} \right) = 0.250$$

$$t = \frac{9.00(\ln 0.250)}{-1.13} \text{ s} = 11.1 \text{ s}$$

continued on next page

$$(c) \quad \frac{dx}{dt} = \left(\frac{mg}{b}\right) \left[1 - \exp\left(-\frac{bt}{m}\right)\right]; \quad \int_{x_0}^x dx = \int_0^t \left(\frac{mg}{b}\right) \left[1 - \exp\left(-\frac{bt}{m}\right)\right] dt$$

$$x - x_0 = \frac{mgt}{b} + \left(\frac{m^2 g}{b^2}\right) \exp\left(-\frac{bt}{m}\right) \Big|_0^t = \frac{mgt}{b} + \left(\frac{m^2 g}{b^2}\right) \left[\exp\left(-\frac{bt}{m}\right) - 1\right]$$

$$\begin{aligned} \text{At } t = 5.54 \text{ s,} \quad x &= 9.00 \text{ kg}(9.80 \text{ m/s}^2) \frac{5.54 \text{ s}}{1.13 \text{ kg/s}} + \left(\frac{(9.00 \text{ kg})^2 (9.80 \text{ m/s}^2)}{(1.13 \text{ m/s})^2}\right) [\exp(-0.693) - 1] \\ x &= 434 \text{ m} + 626 \text{ m}(-0.500) = \boxed{121 \text{ m}} \end{aligned}$$

P6.71 $\sum F_y = L_y - T_y - mg = L \cos 20.0^\circ - T \sin 20.0^\circ - 7.35 \text{ N} = ma_y = 0$

$$\sum F_x = L_x + T_x = L \sin 20.0^\circ + T \cos 20.0^\circ = m \frac{v^2}{r}$$

$$m \frac{v^2}{r} = 0.750 \text{ kg} \frac{(35.0 \text{ m/s})^2}{(60.0 \text{ m}) \cos 20.0^\circ} = 16.3 \text{ N}$$

$$\therefore L \sin 20.0^\circ + T \cos 20.0^\circ = 16.3 \text{ N}$$

$$L \cos 20.0^\circ - T \sin 20.0^\circ = 7.35 \text{ N}$$

$$L + T \frac{\cos 20.0^\circ}{\sin 20.0^\circ} = \frac{16.3 \text{ N}}{\sin 20.0^\circ}$$

$$L - T \frac{\sin 20.0^\circ}{\cos 20.0^\circ} = \frac{7.35 \text{ N}}{\cos 20.0^\circ}$$

$$T(\cot 20.0^\circ + \tan 20.0^\circ) = \frac{16.3 \text{ N}}{\sin 20.0^\circ} - \frac{7.35 \text{ N}}{\cos 20.0^\circ}$$

$$T(3.11) = 39.8 \text{ N}$$

$$T = \boxed{12.8 \text{ N}}$$

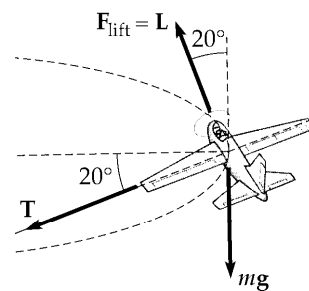


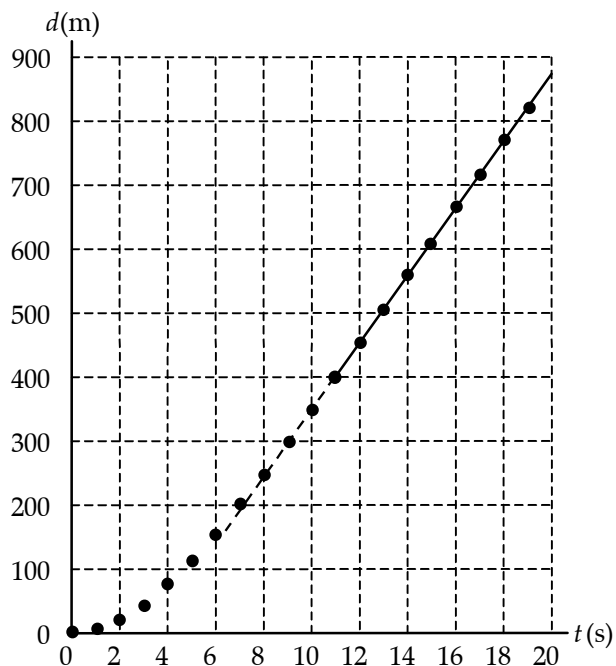
FIG. P6.71

P6.72

(a)

$t(\text{s})$	$d(\text{m})$
1.00	4.88
2.00	18.9
3.00	42.1
4.00	73.8
5.00	112
6.00	154
7.00	199
8.00	246
9.00	296
10.0	347
11.0	399
12.0	452
13.0	505
14.0	558
15.0	611
16.0	664
17.0	717
18.0	770
19.0	823
20.0	876

(b)



- (c) A straight line fits the points from $t = 11.0 \text{ s}$ to 20.0 s quite precisely. Its slope is the terminal speed.

$$v_T = \text{slope} = \frac{876 \text{ m} - 399 \text{ m}}{20.0 \text{ s} - 11.0 \text{ s}} = \boxed{53.0 \text{ m/s}}$$

*P6.73

$v = v_i - kv$ implies the acceleration is

$$a = \frac{dv}{dt} = 0 - k \frac{dx}{dt} = -kv$$

Then the total force is

$$\sum F = ma = m(-kv)$$

The resistive force is opposite to the velocity:

$$\boxed{\sum \mathbf{F} = -km\mathbf{v}}$$

ANSWERS TO EVEN PROBLEMS

P6.2 215 N horizontally inward

P6.12 $2.06 \times 10^3 \text{ rev/min}$

P6.4 $6.22 \times 10^{-12} \text{ N}$

P6.14 (a) $\sqrt{R\left(\frac{2T}{m} - g\right)}$; (b) $2T$ upward

P6.6 (a) 1.65 km/s; (b) $6.84 \times 10^3 \text{ s}$

P6.16 (a) 1.33 m/s^2 ; (b) 1.79 m/s^2 forward and 48.0° inward

P6.8 $0.966 g$

P6.10 (a) $(-0.233 \hat{\mathbf{i}} + 0.163 \hat{\mathbf{j}}) \text{ m/s}^2$; (b) 6.53 m/s ;

P6.18 8.88 N

(c) $(-0.181 \hat{\mathbf{i}} + 0.181 \hat{\mathbf{j}}) \text{ m/s}^2$

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- P6.20** (a) 8.62 m; (b) Mg downward;
(c) 8.45 m/s^2 , Unless they are belted in,
the riders will fall from the cars.
- P6.22** 15.3 m/s Straight across the dashboard to
the left
- P6.24** 0.527°
- P6.26** (a) 1.41 h; (b) 17.1
- P6.28** $\mu_k = \frac{2(vt - L)}{(g + a)t^2}$
- P6.30** (a) $2.38 \times 10^5 \text{ m/s}^2$ horizontally inward
 $= 2.43 \times 10^4 g$; (b) 360 N inward
perpendicular to the cone;
(c) $47.5 \times 10^4 \text{ m/s}^2$
- P6.32** (a) 6.27 m/s^2 downward; (b) 784 N up;
(c) 283 N up
- P6.34** (a) 53.8 m/s; (b) 148 m
- P6.36** 1.40
- P6.38** -0.212 m/s^2
- P6.40** see the solution
- P6.42** 36.5 m/s
- P6.44** (a) 0.980 m/s; (b) see the solution
- P6.46** (a) $7.70 \times 10^{-4} \text{ kg/m}$; (b) 0.998 N;
(c) The ball reaches maximum height 49 m.
Its flight lasts 6.3 s and its impact speed is
27 m/s.
- P6.48** (a) see the solution; (b) 81.8 m; (c) 15.9°
- P6.50** 0.835 rev/s
- P6.52** (a) $mg - \frac{mv^2}{R}$; (b) $v = \sqrt{gR}$
- P6.54** (a) 2.63 m/s^2 ; (b) 201 m; (c) 17.7 m/s
- P6.56** (a) 106 N; (b) 0.396
- P6.58** (a) $m_2 g$; (b) $m_2 g$; (c) $\sqrt{\left(\frac{m_2}{m_1}\right)gR}$
- P6.60** 62.2 rev/min
- P6.62** 2.14 rev/min
- P6.64** (a) $v = \sqrt{\pi R g}$; (b) $m\pi g$
- P6.66** (a) 8.04 s; (b) 379 m/s; (c) 1.19 cm/s;
(d) 9.55 cm
- P6.68** (a) either 70.4° or 0° ; (b) 0°
- P6.70** (a) 78.3 m/s; (b) 11.1 s; (c) 121 m
- P6.72** (a) and (b) see the solution; (c) 53.0 m/s